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14. ABSTRACT <p>Large, high-performance space vehicles will require test-validated finite element models for accurate response predictions. The models of these precision spacecraft must be valid to a higher frequency range, referred to as the mid-frequency range, which is characterized by high modal density. The product of this research project is a complete finite element model validation procedure for the mid-frequency range, or any system with high modal density. The procedure includes sensor placement, analytical model reduction, test-analysis correlation, and finite element model updating. The new approach is totally independent of modal analysis. The results of this research are significant because current state-of-the-art techniques based on modal properties do not work for the mid-frequency range due to high modal density. The new approach differs from past work because it is based on principal components of the system frequency response, and frequency band averaging. The principal components are less sensitive to modeling errors and uncertainties, and can be truncated based on system energy. Frequency band averaging of response allows model uncertainty to be included in the validation process.</p>					
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## EXECUTIVE SUMMARY

Large, high-performance space vehicles being considered by the Air Force will need extremely low level or "ultra-quiet" on-orbit vibration environments. Mission success will require vibration test-validated finite element models of spacecraft and subsystems for accurate response predictions. The aerospace community has traditionally relied on modal analysis and finite element methods for dynamic analysis in the low frequency range. At low frequencies, a relatively small number of modes with widely spaced frequencies can be used to capture system behavior. In contrast, high-performance spacecraft require models that are valid to a higher frequency range for accurate predictions. This higher frequency band, lying between the low frequency range of modal analysis and the high frequency region of statistical energy analysis, is referred to as the mid-frequency range. The corresponding short wavelength vibration patterns require a fine spatial resolution, resulting in a large number of densely packed modes.

The state-of-the-art in finite element model (FEM) validation is based on modal analysis. The validation process is comprised of several activities, such as determining fidelity-to-data, uncertainty quantification, and predictive accuracy. Determining fidelity-to-data is a component of fundamental importance, and the focus of this project. It is the process of comparing test and analysis predictions, also called test/analysis correlation, and then determining optimum changes in parameters that will update, or tune the model. The accuracy of the FEM is determined by comparing test and analysis modal parameters. Frequencies are compared directly, while the corresponding mode shapes are compared using various metrics. The metrics most often employed are the orthogonality and cross-orthogonality of the modes with respect to a reduced analytical mass matrix.

Over the years, there has been a great deal of success using the modal approach for model validation in the low frequency range because the modes are widely spaced in frequency. However, the approach quickly breaks down on many levels in the mid-frequency range due to the high modal density. Analytically, it becomes impossible to identify and select dynamically important modes for the target mode set. Experimentally, the high modal density makes it very difficult to accurately extract modes from the test data. Noise and errors in the test modes, combined with the high modal density, produce a coupling sensitivity between the test and analysis mode shapes. This sensitivity makes orthogonality and cross-orthogonality metrics useless for matching test and analysis mode shapes for subsequent finite element model updating. Systematic validation techniques that worked well for small numbers of target modes turn to disaster for high modal density.

The overall goal of this project was to leverage the knowledge base produced by researchers and practitioners of modal based finite element model validation, and develop a new systematic approach for FEM validation for systems with high modal density.

***The first research accomplishment was the development of a frequency response based sensor placement method.*** The sensor placement scheme for vibration testing is based on the principal components of the frequency response data matrix. The method is a generalization of a modal based technique called Effective Independence (EfI). Sensors are placed to maintain the linear independence of the principal components. This maintains the dynamically important



information in the frequency response and the overall system energy within the frequency range of interest. The frequency response data automatically accounts for input location and damping. Two example applications were investigated, one with low modal density, and the other with high modal density. The results showed that the frequency and modal based EfI methods provide comparable sensor configurations for systems with low damping and well separated modal frequencies. As damping levels increased, the frequency based EfI approach automatically skews the sensor locations toward the input locations. Both examples demonstrated that the frequency based EfI technique automatically accounts for how the selected inputs excite the structure, and thereby eliminates the need for the user to identify the target modes. In the high modal density example, it was not only impossible to select and distinguish the appropriate target modes, there were so many of them that there were not enough sensors available to render them linearly independent. This makes modal EfI impossible to use, and frequency EfI, based on principal directions, a valuable tool for sensor placement in systems with high modal density.

*The second research accomplishment was the development of a technique for the reduction of the finite element model to the sensor degrees of freedom.* Reduction is needed because the vibration test does not measure response at all finite element model degrees of freedom, just at the sensor locations. The frequency domain model reduction scheme is based on the principal components of the frequency response data matrix. The new model reduction strategy was created to accompany the frequency domain Effective Independence sensor placement technique. The frequency domain reduction method eliminates the difficulties associated with modal based reduction and correlation techniques in the mid-frequency range due to high modal density. Model based test-analysis correlation metrics that utilize the frequency response and the reduced model are employed to replace the use of standard modal based metrics. The frequency domain model reduction uses a transformation matrix that relates the omitted degrees of freedom to the sensor locations through the dominant principal directions of the frequency response data matrix. The resulting transformation matrix is independent of frequency, while remaining valid over the entire frequency range of interest. The two previously mentioned examples were used to demonstrate the model reduction and correlation techniques. The low modal density application showed that the accuracy of the reduction is dependent on the number of retained principal directions. It also illustrated that the new reduction method does not suffer from sensitivity to the system's constrained dynamics, as was demonstrated with an exact dynamic reduction. The high modal density example was included to demonstrate a successful application where traditional modal based reduction and correlation techniques fail due to problems associated with high modal density.

*The third research accomplishment was the development of a new correlation metric based on frequency response in which the metric error can be directly related to error in the system response.* Accepted modal based techniques for comparing FEM and Test data for test/analysis correlation and subsequent FEM updating are impossible to use in the high modal density mid-frequency regime. A new approach was developed for comparing test and analysis representations using frequency band averaging of the output power spectral densities (PSD), with the central frequency of the band running over the complete frequency range of interest. The results of this computation were interpreted in several different ways, but the immediate physical connection is that it produces the mean square response, or energy, of the system to

random input limited to the averaging frequency band. As the averaging band gets smaller, the comparison is more critical until at a zero-width averaging band, it is just a point-wise comparison of the Test and FEM PSDs. The more accurate the FEM, the more narrowly, or more concentrated, with respect to frequency, the input can be applied and still maintain accuracy in the predictions. The averaged response curves can be compared on a point-wise basis, or they can be compared within a running frequency band. Frequency band averaging can be used to quantify the accuracy of the FEM with respect to the Test data in a physically meaningful way, in contrast with past modal techniques using metrics with no direct connection to FEM prediction errors. The new approach is physically motivated and more reasonable than a direct comparison of frequency response, and is also more quantitative than the usual visual comparison of test and analysis frequency response. Several example applications were used to demonstrate the technique.

*The final research accomplishment was the development of a finite element model updating strategy based on the new frequency domain correlation metric.* The new method uses an output error approach for updating the finite element model. The output error metric is based on frequency band averaging of the output power spectral densities as discussed in the summary of the previous task. The use of spectral densities has several advantages over using frequency response directly, such as the ability to easily include data from all inputs at once, and the fact that the metric is real. Finite element model mass and stiffness matrix sensitivities with respect to selected design variables are computed within a finite element code. The size of the optimization problem is significantly reduced, and the stability of the algorithm is improved by transforming to modal space. If needed, transforming to the principal component space of the frequency response data matrix can further reduce the size of the problem. The optimization is based on the computation of the gradient of the cost function with respect to the dimensionless design variables. The explicit computation of the Hessian was also implemented. The new updating technique was successfully applied to several low, and high modal density examples. It was shown that the averaging process reduces the sensitivity of the optimization to resonances that plagues many output error model updating approaches. Averaging also reduces the effects of noise in the test data.

The product of this project is a complete and systematic procedure for finite element model validation for the mid-frequency range that is totally independent of modal analysis. The proposed research is very significant to the Air Force because current state-of-the-art techniques based on modal properties do not work for high-performance spacecraft in the mid-frequency range due to high modal density.

**The work performed during this project was conducted by:**

Dr. Daniel C. Kammer, Professor  
Mr. Sonny A. Nimityongskul, Research Assistant

**Project research resulted in four publications:**

Nimityongskul, S., and Kammer, D. C., "Frequency Response Based Sensor Placement for the Mid-Frequency Range", 26<sup>th</sup> International Modal Analysis Conference, Orlando, FL, Feb. 2008. Accepted by *Mechanical Systems and Signal Processing*.

Kammer, D. C., and Nimityongskul, S., "Energy Based Comparison of Test and Analysis Response in the Frequency Domain", 26<sup>th</sup> International Modal Analysis Conference, Orlando, FL, Feb. 2008. Submitted to *ASME Journal of Vibration and Acoustics*.

Nimityongskul, S., and Kammer, D. C., "Frequency Domain Model Reduction Based on Principal Component Analysis", 27<sup>th</sup> International Modal Analysis Conference, Orlando, FL, Feb. 2009. Submitted to *Mechanical Systems and Signal Processing*.

Kammer, D. C., and Nimityongskul, S., "Frequency Band Averaging of Spectral Densities for Updating Finite Element Models", 27<sup>th</sup> International Modal Analysis Conference, Orlando, FL, Feb. 2009. Accepted by *ASME Journal of Vibration and Acoustics*.



# A FREQUENCY DOMAIN APPROACH TO PRETEST ANALYSIS, MODEL CORRELATION, AND MODEL UPDATING FOR THE MID-FREQUENCY RANGE

## 1.0 INTRODUCTION

High-performance, precision space vehicles being considered by the Air Force will need extremely low-level on-orbit vibration environments. Mission success will require accurate predictions of on-orbit performance using analytical models that have been validated via vibration tests. The aerospace community has traditionally relied on modal methods and finite element analysis for dynamic predictions in the low frequency range. At low frequencies, a relatively small number of modes with widely spaced frequencies can be used to capture system behavior. In contrast, precision spacecraft require models that are valid to a much higher frequency range for accurate predictions. This higher frequency band, lying between the low frequency range of modal analysis and the high frequency region of statistical energy analysis, is referred to as the mid-frequency range. The corresponding short wavelength vibration patterns result in a large number of densely packed modes. A fine spatial resolution is required in the finite element model (FEM).

The state-of-the-art in FEM validation is based on modal analysis. The validation process is comprised of several activities, such as determining fidelity-to-data, uncertainty quantification, and predictive accuracy. Determining fidelity-to-data is a component of fundamental importance, and the focus of this project. It is the process of comparing test and analysis predictions, also called test/analysis correlation, and then determining optimum changes in parameters that will update, or tune the model. The accuracy of the FEM is determined by comparing test and analysis modal parameters. Frequencies are compared directly, while the corresponding mode shapes are compared using various metrics. The metrics most often employed are the orthogonality and cross-orthogonality of the modes with respect to a reduced analytical mass matrix, called a test-analysis model (TAM). The use of these metrics, and the required values for test/analysis correlation, are dictated by agencies such as NASA and the United States Air Force. Depending on the agency, the requirements are different. For example, the Air Force requires test/analysis frequency errors less than or equal to 3.0%, cross-generalized mass values greater than 0.95, and coupling terms between modes of less than 0.10 in both cross-orthogonality and orthogonality. A significant drawback of these metrics is the fact that they are not physically motivated. There is not a direct connection between the metric values and the corresponding error in the predicted response.

Over the years, there has been a great deal of success using the modal approach for model validation in the low frequency range because the modes are widely spaced in frequency. However, the approach quickly breaks down on many levels in the mid-frequency range due to the high modal density. Analytically, it becomes impossible to identify and select dynamically important modes for the target mode set. Experimentally, the high modal density makes it very difficult to accurately extract modes from the test data. Noise and errors in the test modes, combined with the high modal density, produce a coupling sensitivity between the test and analysis mode shapes. This sensitivity makes orthogonality and cross-orthogonality metrics useless for matching test and analysis mode shapes, which is vital to the subsequent finite element model updating. Also, as the vibrational wavelengths approach the scale of structural

variations, the mode shapes become very sensitive to modeling errors and uncertainties. Systematic validation techniques that worked well for small numbers of target modes turn to disaster for high modal density. Users tend to fall back on trial and error techniques.

The product of this project is a complete and systematic procedure for finite element model validation for the mid-frequency range that is totally independent of modal analysis. The proposed research is very significant to the Air Force because current state-of-the-art model validation techniques based on modal properties do not work for high-performance spacecraft in the mid-frequency range due to high modal density.

## **2.0 SIGNIFICANT RESEARCH ACCOMPLISHMENTS**

### **2.1 Frequency Response Based Sensor Placement**

At low frequencies, a relatively small number of modes with widely spaced frequencies can be used to capture system behavior. Sensor placement for a model validation vibration test is based on a relatively small set of target modes within the frequency range of interest. In contrast, precision spacecraft require models that are valid to a much higher frequency range for accurate predictions. The drawback to using many of the current sensor placement methods in this mid-frequency range is that they are based on the mode shapes of the pretest FEM. Modal based techniques break down in the mid-frequency range due to the high modal density. As frequencies are increased, it becomes increasingly difficult to identify and select the dynamically important modes.

A frequency domain sensor placement scheme for vibration testing was developed during this project based on the principal components of the frequency response data matrix. The method is a generalization of a modal based technique called Effective Independence (EfI). However, instead of placing sensors to maintain the linear independence of the target mode shapes, sensors are placed to maintain the independence of the principal components. This maintains the dynamically important information in the frequency response and the overall system energy within the frequency range of interest. The frequency response data automatically accounts for force input location and damping. Two example applications were investigated, one with low modal density, and the other with high modal density. The results showed that frequency and modal based EfI methods provide comparable sensor configurations for systems with low damping and well separated modal frequencies. As damping levels increased, the new frequency based EfI approach automatically skews the sensor locations toward the input locations. Both examples demonstrated that the frequency based EfI technique automatically accounts for how the selected inputs excite the structure, and thereby eliminates the need for the user to identify the target modes.

The high modal density example considered the placement of triaxial sensors on the Generic Spacecraft (GSC), illustrated in Fig. 1. The GSC is composed of a cubic core, two circular rib stiffened reflectors, and two rectangular photovoltaic (PV) arrays. The GSC's finite element model contained 1191 nodes and 1262 elements. Six inputs in the z-direction (into the paper) were considered for the computation of the frequency response, one on the end of each PV array and two on each reflector. One percent modal damping was assumed. The frequency band of interest was from 50 to 300 Hz, which contained 412 vibrational modes. The modes are close in



frequency, and very localized. Therefore, the model exhibited all the problems associated with the mid-frequency range. Closely spaced modes can be difficult to distinguish in a modal test,

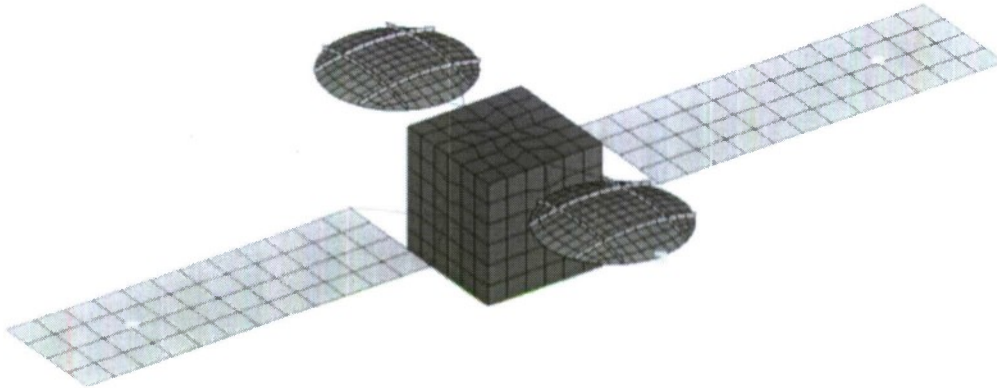


Fig. 1. Generic spacecraft finite element model.

and when coupled with noise, create problems for correlation metrics, such as orthogonality and cross-orthogonality. Modal based sensor placement techniques fail because the number of sensors required for linear independence of the modes is prohibitive. Here, modal based Efl would require a minimum of 138 triaxial sensors to render the mode shapes linearly independent, and as many as 412 triaxial sensors to guarantee independence. This problem lends itself more naturally to the new frequency response based sensor placement technique, because the frequency response is dominated by a relatively small number of principal directions, as compared to the mode shapes.

The analytical frequency response data matrix for the band from 50 to 300 Hz was calculated using a modal expansion. All mode shapes up to 450 Hz were used in the calculation to account for contributions from out-of-band modes. The singular values of the frequency response data matrix were computed and truncated to the top 95% of their total sum, resulting in 121 being retained. This shows that the bulk of the system's energy is captured in 121 principle directions, which is roughly 70% fewer shapes than the number modes contained in the 50 to 300 Hz. frequency band. The new frequency domain Efl triaxial sensor set expansion method was used to obtain 60 triaxial sensor locations. A total of 59 iterations were run to expand the initial sensor set to the final 60 sensor locations. Figure 2 shows the final triaxial sensor positions on the GSC. It is apparent that all of the sensors for this mid-frequency example are clustered on the circular reflectors. This can be attributed to the fact that the majority of the modes excited by the selected inputs in this frequency range have displacements that are highly localized to the reflector surfaces.

In this high modal density example, it was not only impossible to select and distinguish the appropriate target modes, there were so many of them that there were not enough sensors available to render them linearly independent. This makes modal Efl impossible to use. In



contrast, frequency domain Efl, based on principal directions, is a valuable tool for sensor placement in high modal density systems. The theoretical details of this frequency domain sensor placement technique can be found in Ref. [1].

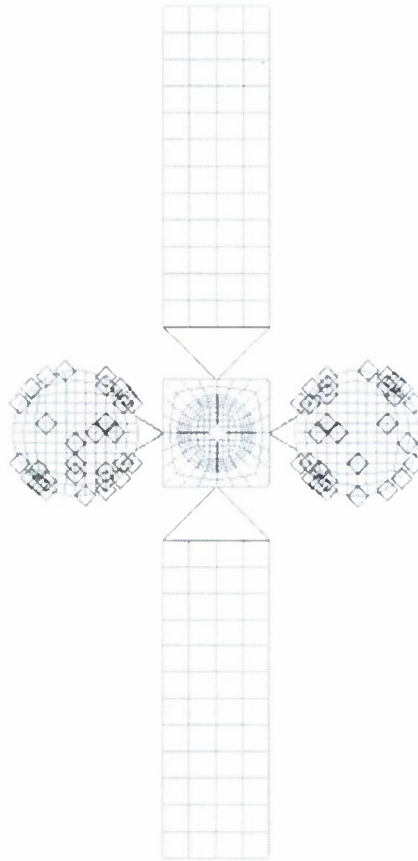


Fig. 2. Triaxial sensor locations selected using frequency domain Efl.

## 2.2 Finite Element Model Reduction to Sensor Degrees of Freedom

The metrics most often used for test-analysis correlation in the low frequency range are orthogonality and cross-orthogonality of the mode shapes with respect to an analytical mass matrix that has been reduced to the sensor degrees of freedom. The reduced mass matrix is referred to as a test-analysis model, or TAM. Government agencies such as NASA and the United States Air Force require the use of these metrics with specific criteria for determining modeling accuracy, as mentioned previously. However, with high modal density systems, it becomes increasingly difficult to select a set of dynamically important target modes. In addition, test noise and model errors combined with mode coupling produce sensitivity between test and analysis mode shapes. This sensitivity makes modal based orthogonality correlation metrics useless for matching test and analysis modes in the mid-frequency range.

To avoid the use of modal based metrics, test-analysis correlation metrics have been developed over the years that directly compare test and FEM frequency response. Many of the metrics do not require the use of a reduced FEM representation. However, these metrics only give

information regarding the accuracy of the FEM at the measured degrees of freedom. There is no check on the consistency of the rest of the FEM with respect to the measured response. In contrast, the inclusion of a TAM representation within an equation of motion based metric provides additional information about the FEM in the form of a consistency check between the unmeasured and measured degrees of freedom. Therefore, frequency domain metrics have been developed to determine the consistency of the test frequency response data with respect to the analytical impedance matrix reduced to the sensor locations, which can be considered a frequency based TAM.

The reduced model must accurately represent the dynamics of the full system in the frequency range of interest. Many model reduction techniques have been developed for test-analysis correlation in the low frequency range. The most popular technique is known as static or Guyan reduction. Guyan reduction uses the FEM static equations of motion to derive a set of static shapes that form a reduced basis for the FEM displacement. If sensor locations associated with high mass-to-stiffness ratio degrees of freedom can be selected, the static TAM can produce very accurate representations at lower frequencies. However, static reduction fails in the mid-frequency range because the neglected frequency dependent terms become more dominant. The shortcomings of the static reduction led to the development of other advanced model reduction techniques, such as Modal, Hybrid, SEREP, and IRS [2]. The IRS reduction method extends the static reduction by approximating the frequency dependent terms that the static TAM neglects. The Modal, Hybrid, and SEREP reduction methods all use a set of target modes to form a transformation matrix to reduce the FEM to the sensor degrees of freedom. As before, the modal based reduction techniques break down in the mid-frequency range because too many sensors are required to account for the high modal density. An exact dynamic reduction has also been used to create reduced models that are exact representations of the full model. However, this process is very computationally intensive because a new transformation matrix is required at each frequency point. Additionally, dynamic reduction has difficulties with sensitivity due to the constrained dynamics, which limits its effectiveness in model updating.

This research project developed a new frequency domain model reduction scheme based on the principal components of the frequency response data matrix. The new model reduction strategy was created to compliment the frequency domain Effective Independence sensor placement technique discussed previously. The frequency domain reduction method eliminates the difficulties associated with modal based reduction and correlation techniques in the mid-frequency range due to high modal density. Model based test-analysis correlation metrics that utilize the frequency response and the reduced impedance matrix are employed to replace the use of modal based metrics. The TAM impedance matrix can be used in FEM-TAM, or test-analysis correlation metrics to check for TAM accuracy, or fidelity of the FEM with respect to the test data. The frequency domain model reduction uses a transformation matrix that relates the omitted degrees of freedom to the sensor locations through the dominant principal directions of the frequency response data matrix. The resulting transformation is independent of frequency, while remaining valid over the frequency range of interest. The accuracy of the model reduction is dependent on retaining enough principal directions to adequately describe the system dynamics in the frequency band.

The orthogonality metric using the reduced model is calculated by taking the product of the TAM impedance matrix and the frequency response data at each frequency point given by

$$\begin{bmatrix} Z_{TAM_{aj}} \\ Z_{TAM_{nj}} \end{bmatrix} \hat{h}_{saj} = \begin{bmatrix} O_{aj} \\ O_{nj} \end{bmatrix} = \begin{bmatrix} I_a \\ 0_{na} \end{bmatrix} \quad (1)$$

The frequency response data can be taken from either the full FEM, for FEM-TAM correlation, or from experimental data for test-analysis correlation. The experimental, or FEM frequency response data,  $\hat{h}_{saj}$  in Eq. (1), only contains rows corresponding to sensor locations and columns corresponding to the input locations. It should be noted that for FEM-TAM correlation, Eq. (1) does not hold exactly unless the TAM impedance matrix is an exact representation of the FEM. In test-analysis correlation, Eq. (1) will never be exactly satisfied.

The two previously mentioned examples were used to demonstrate the model reduction and FEM/TAM correlation techniques. The low modal density example was used to show that the accuracy of the reduction is dependent on the number of retained principal directions. It also showed that new the reduction method does not suffer from sensitivity to the system's constrained dynamics, as was demonstrated with the exact dynamic reduction. The term "constrained dynamics" refers to the natural frequencies corresponding to the FEM constrained at the sensor locations. The sensitivity occurs because the exact reduction procedure uses the inverse of the partition of the FEM impedance matrix that corresponds to the degrees of freedom that are reduced out of the representation.

The Generic Spacecraft was again used as a high modal density example to demonstrate an application in which traditional modal based reduction and correlation techniques fail due problems associated with the high modal density. The frequency band of interest was 100 to 200 Hz., which contains 158 vibrational modes. The majority of the modes in this frequency range are localized on the reflector surfaces; so two inputs were placed on each reflector to excite these modes. The analytical frequency response was generated using all the mode shapes up to 450 Hz. The residual flexibility correction was added to the FRF data to account for the response due to the omitted modes. The top 150 principal directions from the FRF data matrix in the 100 to 200 Hz. frequency range were retained and used to select 200 uniaxial sensor locations using the FEfl sensor placement technique discussed in the previous section. The retained principal directions account for over 99.99% of the total energy of the system. The model reduction used roughly 78% fewer principal directions as compared to the number of modes used to create the frequency response.

Figure 3 illustrates some selected terms from the FEM-TAM correlation for the  $O_{aj}$  metric in Eq. (1). The remaining terms are not shown, but yielded similar results. The diagonal terms of the  $O_{aj}$  metric easily fall between 0.9 and 1.1 for each point in the frequency band of interest, and the off-diagonal terms are all less than 0.1. The reduced model provides an accurate representation of the FEM in the frequency range of interest. The principal direction transformation accurately reduced the size of the model by nearly 95%, while using only a single transformation matrix for the entire frequency band of interest. The theoretical details of this frequency domain model reduction technique can be found in Ref. [2].



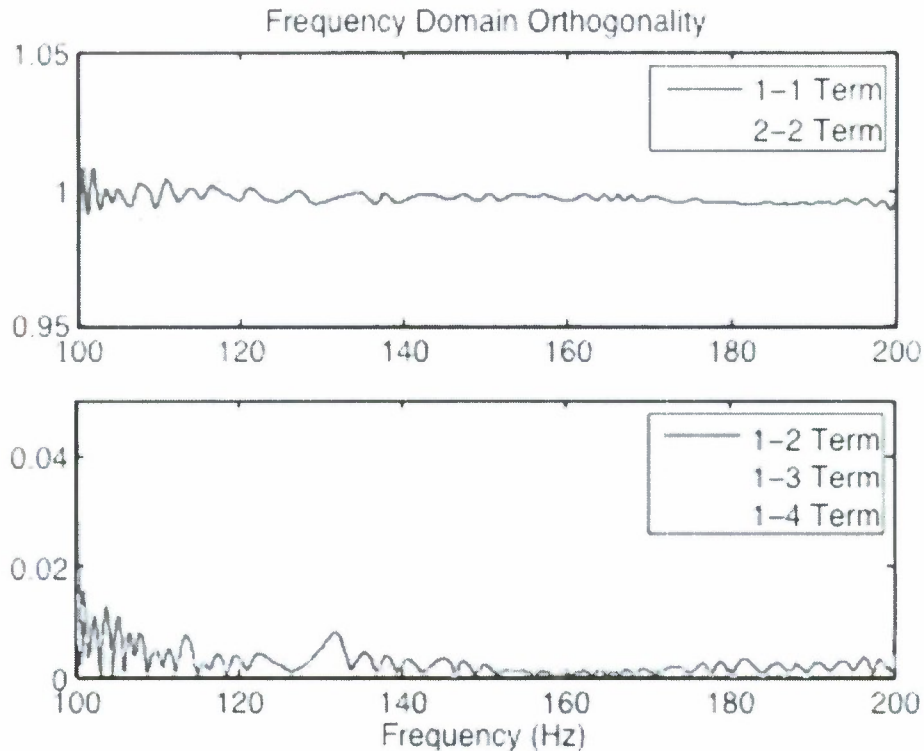


Fig. 3. Selected terms from orthogonality metric for Generic Spacecraft.

### 2.3 Test-Analysis Correlation using Frequency Band Averaging

Accepted modal based techniques for comparing finite element model and test data for test/analysis correlation and subsequent model updating are impossible to use in the high modal density mid-frequency regime. During this project, a new approach was developed for comparing test and analysis representations using frequency based response data, instead of modal parameters. The new method uses Frequency Band Averaging (FBA) of the output power spectral densities (PSD), with the central frequency of the band running over the complete frequency range of interest. The results of this computation can be interpreted in several different ways, but the immediate physical connection is that it produces the mean square response, or energy, of the system to random input limited to the averaging frequency band. As the averaging band gets smaller, the comparison is more critical until at a zero-width averaging band, it is just a point-wise comparison of the Test and FEM power spectral densities. The more accurate the FEM, the more narrowly, or more concentrated, with respect to frequency, the input can be applied and still maintain accuracy in the predictions.

The averaging process is consistent with the averaging done in statistical energy analysis for stochastic systems. The FEM will always contain model uncertainties, and can be considered as a single model from within a random population. The effects of model uncertainty are especially important in the mid-frequency range, and the proposed averaging approach for test/analysis correlation accounts for it. The averaged response curves can be compared on a point-wise basis, or they can be compared within a running frequency band. This later approach, called

Frequency Band Correlation (FBC), can be used to determine the minimum error between the FEM and Test responses within an allowable frequency band about each frequency point within the overall frequency range of interest. This technique allows the user to determine trends in frequency shifts between FEM and Test response, and is analogous to the allowable frequency errors in conventional modal frequency correlation. The proposed method can be applied to the response at individual sensors, the sums of subsets of sensors corresponding to different substructures, or the total response sum related to signal energy at the sensors. Any type of output can be used, as well as FEM mass weighting for kinetic energy. A FEM/Test mean square shape comparison can also be incorporated using a modal assurance criterion type comparison. In contrast to modal based comparisons, the new approach is physically motivated, and it is more reasonable than a direct comparison of frequency response. It is also more quantitative than the usual visual comparison of test and analysis frequency response. Several simple examples were studied during the course of the project.

The Generic Spacecraft was again used as a more detailed model that exhibited high modal density. The initial analytical representation was considered as the Test article. There are 351 Test modes with frequencies in the frequency range of interest, from 100-300 Hz. The pretest FEM was generated by modifying several of the photovoltaic array and reflector physical, and material properties. For example, some of the physical properties, such as sandwich core thicknesses, were increased or decreased by 9%; while some of the material properties were selected from zero mean normal distributions. The elastic modulus for one of the reflectors was increased by 30%, while the other was increased by 9%. The mode overlap factors for both the Test and FEM representations, assuming 1% modal damping, range between 5 and 25 in the 100-300 Hz. range. Sensors for a simulated vibration test were selected at 102 locations based on 6 inputs (2 on each reflector and one on each of the array tips), and the pretest FEM frequency response data sampled at 0.1 Hz. from 1 to 450 Hz.

In order to reduce the amount of data to be considered when comparing the FEM and Test responses, the response was added over all the sensors at each frequency data point. This gives a measure of the total response signal energy in the structure at the sensor locations due to a uniform random input limited to the corresponding averaging band. If the averaging band were limited to the data frequency resolution, the summed PSD responses would produce a measure of the energy due to a pure tone. Figure 4 illustrates a comparison between the summed Test and FEM PSDs. The corresponding modal density can be seen at the bottom of the plot. Even though the density is high, there are distinct overall response features that span many modes. In some cases, there is just a shift of the feature in frequency from Test to FEM. In other cases, there is significant difference in magnitude, and others, there is a shift in frequency and a significant difference in response level. It can be seen from the figure that there are large differences between the Test and FEM response feature magnitudes in the frequency bands 110-120 Hz., and 190-240 Hz. Relative to the maximum Test value within the frequency range of interest, there are errors of up to 140%. Therefore, on a point-wise basis, the FEM and Test representations do not correlate very well.

In an attempt to correlate FEM and Test data accounting for model uncertainty, FBA is applied to the PSD data sets. Figure 5 shows the sum of the mean square responses, or response energy, at the sensors for a 6% averaging band. Figure 6 illustrates the corresponding FEM/Test error relative to the maximum Test response within the frequency range of interest. The averaging process reduces the error dramatically, but it is still significant. Frequency band correlation

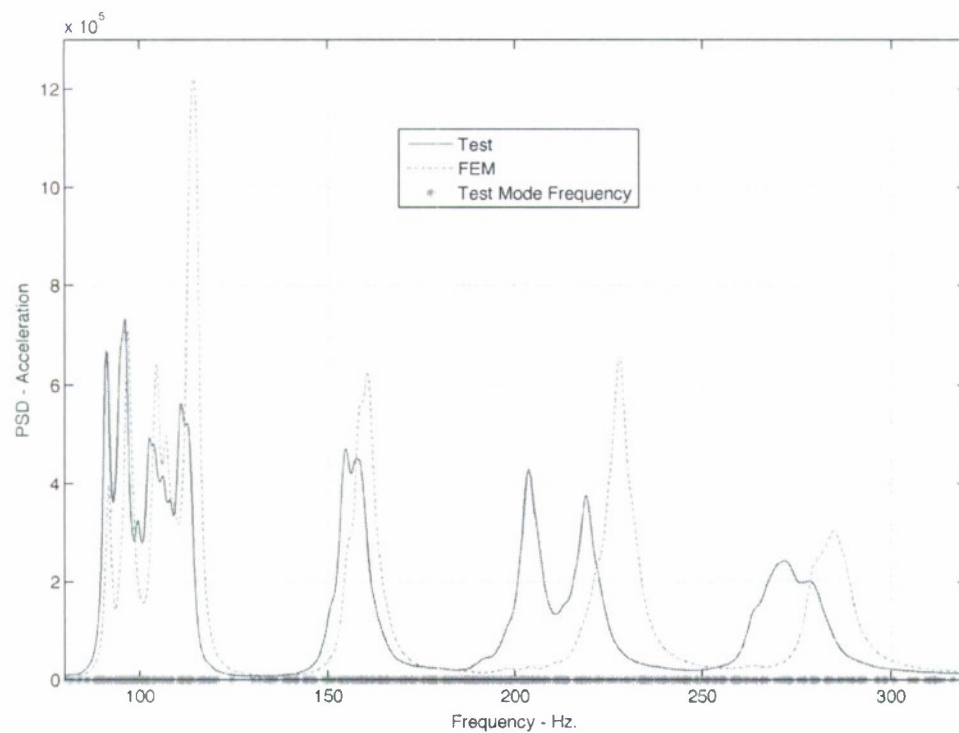


Fig. 4. Test and FEM PSDs for GSC.

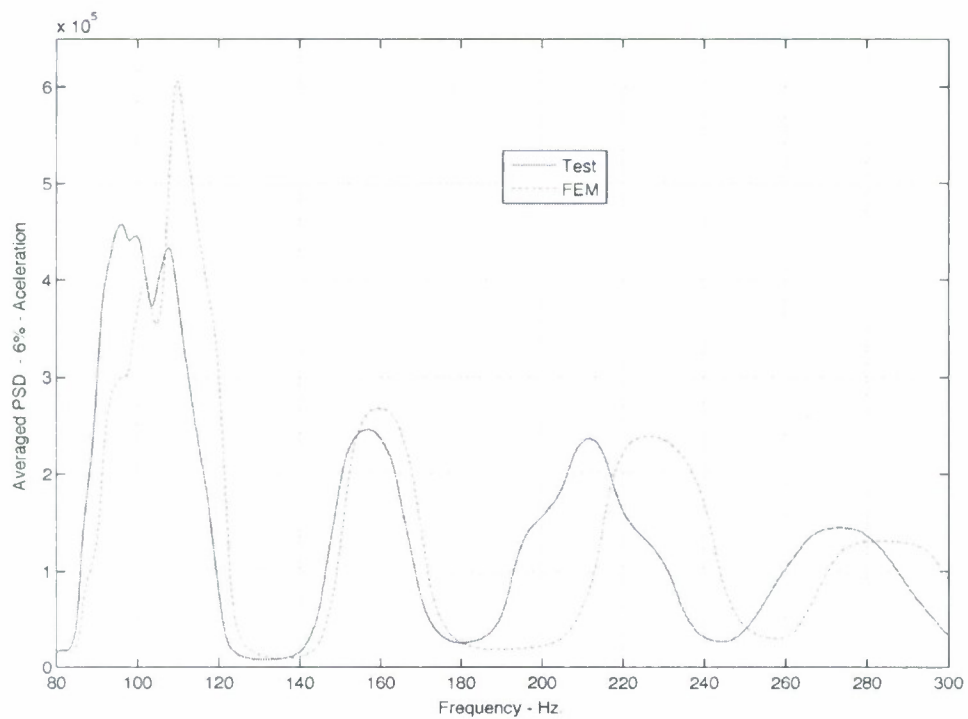


Fig. 5. Test and FEM PSDs for GSC averaged over a 6% frequency band.



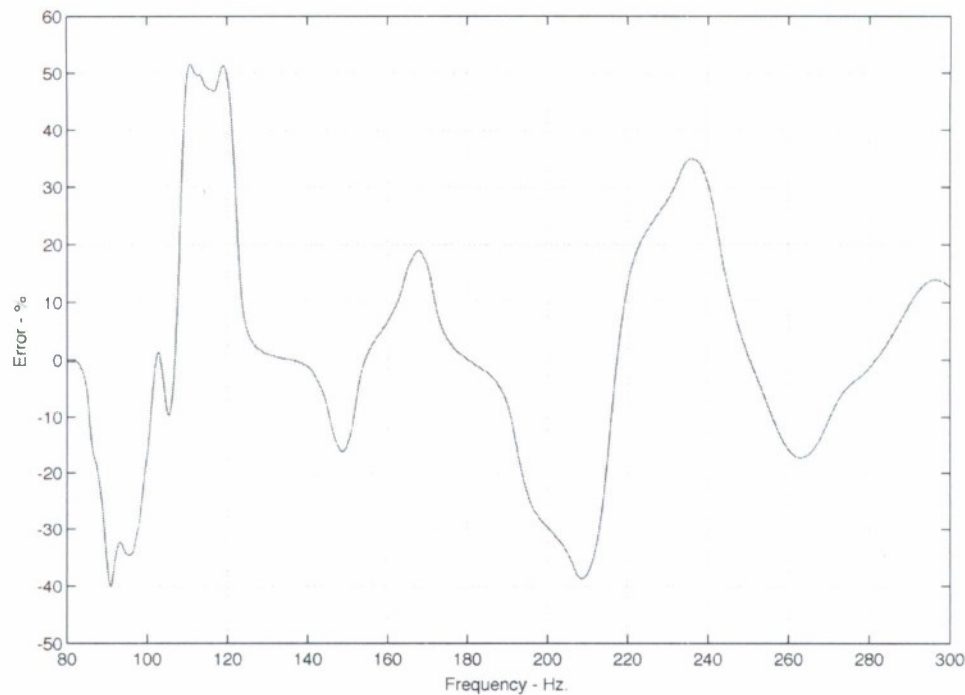


Fig. 6. GSC FEM/Test error relative to maximum Test response - 6% FBA.

applied to the averaged data for 3%, 6%, and 10% correlation bands is illustrated in Fig. 7. A correlation band of 10% reduces the FEM/Test error to less than 10% within the 100-300 Hz. frequency range for the 6% averaged data. Using an averaging band of 15% reduces the FEM/Test response error to less than 20% over most of the desired frequency range. This amount of error is most likely unacceptable for most applications. The error can be reduced to less than 10% by applying FBC with a 6% correlation band, instead of the 10% band required for the 6% FBA data. Increasing the averaging band to 20% does little to reduce the error further. Therefore, model uncertainty alone probably cannot be used to explain the differences in the FEM and Test total energy responses within the frequency range of interest.

Based on the correlation results presented using the proposed FBA and FBC approaches, it can be said that the total energy response of the FEM agrees with the energy response of the Test to within 20% for uniform random input limited to a 15% running frequency band. Or, it can be said that 15% model uncertainty can be used to explain the difference between the FEM and Test responses to within 20%. In addition, for 15% FBA, the FEM predicts the Test mean square response, or energy, to less than 10% error within a 6% frequency correlation band about each frequency point. The advantage of using the FBA and FBC methods of model correlation is that specific statements, such as these, can be made regarding the accuracy of the FEM. In contrast, no such statements can be made based on the modal correlation results. While a detailed analysis would have to be performed to accurately quantify the modal frequency modeling uncertainty, a modal correlation analysis indicated an rms error of 8.20%. Assuming this as the model uncertainty level for FBA did not result in a good FEM/Test agreement, and even assuming values of up to 20% model uncertainty did not provide reasonable correlation. This indicates that the FEM must be updated using an appropriate technique to reduce the frequency

shifts and magnitude differences between FEM and Test response features. The theoretical details of this new test/analysis correlation technique can be found in Ref. [3].

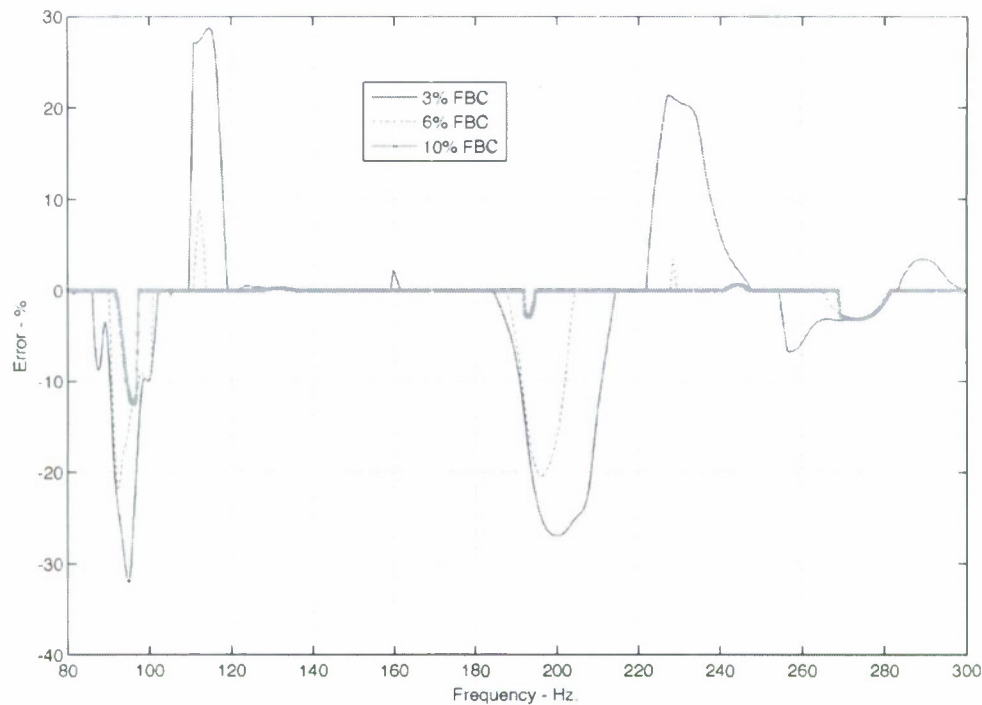


Fig. 7. GSC FEM/Test relative errors using FBC on 6% averaged data.

## 2.4 Finite Element Model Updating using Frequency Band Averaging

There has been a great deal of success using the modal approach for test/analysis correlation and model updating in the low frequency. The approach quickly breaks down on many levels in the mid-frequency range. Noise and errors in the test modes, combined with the high modal density, produce a coupling sensitivity between the test and analysis mode shapes. This sensitivity makes modal orthogonality metrics useless for matching test and analysis mode shapes for subsequent finite element model updating. Systematic validation techniques that worked well for small numbers of target modes turn to disaster for systems with high modal density. Consequently, in the mid-frequency range, modal approaches should be avoided by directly using frequency-based responses for FEM validation.

A great number of researchers have studied model updating and damage detection in the frequency domain. Methods can be loosely categorized based on the error definition used in the penalty/objective function, equation error, or output error. Both approaches are based on the frequency domain equation of motion. In equation error methods, sensitivities of the FEM impedance matrix with respect to the design variables must be computed. An advantage of this approach is that the impedance functions are usually smooth functions of the design variables, and sometimes even linear. This leads to more robust optimization analysis. One of the drawbacks of equation error approaches is that not all FEM degrees of freedom are measured during the vibration test. The FEM impedance matrix must be reduced to the sensor degrees of freedom. It can be shown that the reduction process can produce non-smooth reduced

impedance functions that have sharp peaks at the resonances of the structure in which the sensor degrees of freedom fixed. This can lead to instabilities in the optimization of the design variables.

Output error techniques minimize the error between the measured data and the analytical prediction of that data. In many studies using this approach, the drawback has been due to the fact that the frequency response based error metrics incorporated were not physically motivated. There is no direct connection between metric error and the error in the FEM predicted response. The advantage of the output approach is that no model reduction is required. However, these techniques require the minimization of a nonlinear objective function that can lead to associated convergence problems and large computational effort. When the error metric is based on direct comparison of frequency response to maintain the physical connection, it has been found that minimization of the objective functions can lead to instabilities near resonances.

This project produced a new output error approach for finite element model updating using the new test/analysis correlation metric discussed in Section 2.3. The metric is based on frequency band averaging of the output power spectral densities with the central frequency of the band running over the complete frequency range of interest. This maintains a direct connection to physical response. The use of spectral densities has several advantages over using frequency response directly, such as the ability to easily include data from all inputs at once, and the fact that the metric is real. It was also shown that the averaging process reduces the sensitivity of the optimization due to resonances that plagues many output error model updating approaches. Averaging also reduces the effects of noise in the test data.

The mean square shape of the structure at frequency  $\omega_i$  can be defined as the vector

$$\Psi_i = \frac{1}{n_i} \sum_{j=i-l}^{i+l} \text{diag}(H_j H_j^*) \quad (2)$$

in which  $H_j$  is the frequency response matrix at frequency  $\omega_j$ , and  $n_i$  is the number of data points in the frequency averaging band. The fractional error in the mean square shape can then be defined in the form

$$\varepsilon_i = \frac{1}{\beta} (\Psi_i - \Psi_i^e) = \tilde{\Psi}_i - \tilde{\Psi}_i^e \quad (3)$$

where  $\tilde{\Psi}_i$  and  $\tilde{\Psi}_i^e$  represent the FEM and test mean square shapes, respectively, normalized with respect to the maximum value of the test mean square shape,  $\beta$ , within the frequency band of interest. This normalization method prohibits the situation in which frequency data points with small test response dominate the error in mean square shape. Following a standard multidisciplinary design optimization procedure, the objective function to be minimized is cast in the form

$$J = \sum_{i=1}^{n_f} \varepsilon_i^T W_\varepsilon \varepsilon_i + \alpha (\Delta D^T W_D \Delta D) \quad (4)$$



in which  $n_f$  is the number of frequency data points included in the optimization, and  $\Delta D$  is a vector of changes in the normalized design variables. The minimization of  $J$  is subject to the design variable interval constraints  $d_r^L \leq d_r \leq d_r^U$ .

Minimization of the first term in the objective function in Eq. (4) will minimize the error in the FEM, while minimization of the second term will attempt to maintain small changes in the design variables. This assumes that the FEM is initially close to being an accurate representation of the test data. The coefficient  $\alpha$  in Eq. (4) allows the importance of minimization of model error relative to minimization of design variable changes to be adjusted. If the model and test data have no errors, the update can be made deterministically, in which case  $W_\epsilon$  and  $W_D$  are diagonal weighting matrices used to adjust the relative importance of minimizing error at one sensor location versus another, and the relative importance of minimizing the change in one design variable versus another, respectively. If the analyst is confident in the value of the nominal design variable, the corresponding weighting value would be high relative to others. In reality, uncertainty will exist in both the FEM and test data. In this case, the weighting matrices,  $W_\epsilon$  and  $W_D$ , are taken as the inverses of the covariance matrices for the response measurement error and the initial parameter estimates, respectively. Theoretical details of the new model updating approach can be found in Ref. [4].

Several simple examples were investigated during this project to demonstrate the effective use of frequency band averaged spectral densities to accurately update a finite element model. The Generic Spacecraft example discussed in Section 2.3 was also used in the model updating investigation. It represents a more realistic application with high modal density. The frequency range of interest is 0.0 to 300.0 Hz. Sixteen system parameters were arbitrarily selected as design variables as listed in Table 1.

Table 1. Design variables for GSC.

	Description	Parameter	Error - %
1	PV Array 1	Thickness - T2	-9.09
2		Thickness - T3	-9.12
3	PV Array 2	Thickness - T2	-9.09
4		Thickness - T3	0.00
5	Core Stiffeners	Dimension 3	0.00
6		Dimension 4	0.00
7	Connect Beams	Dimension 3	-9.06
8		Dimension 4	11.08
9	Core Panels	Thickness - T2	0.00
10		Thickness - T3	0.00
11	Reflector 1	Thickness	0.00
12	Reflector 2	Thickness	0.00
13	PV Array 1	Elastic Modulus	-2.81
14	PV Array 2	Elastic Modulus	12.94
15	Reflector 1	Elastic Modulus	9.44
16	Reflector 2	Elastic Modulus	-5.88

Nominal FEM and simulated Test frequency responses were calculated using modal expansion and all 541 modes from 0.0 to 450.0 Hz, 1.5 times the frequency range of interest. Velocity response was calculated at 146 sensor locations based on 6 inputs (2 on each reflector and one on each of the array tips). Figure 8 illustrates nominal FEM and Test velocity PSDs summed over all sensors within the frequency range of interest. There are significant differences in response in the 150.0 to 300.0 Hz. region. Computing the Euclidean norm of the vector  $\{\partial \epsilon_i / \partial d_r\}$  at each frequency in the range of interest indicated that design variables 7, 8, 11, 12, 15, and 16 from Table 1 dominate the measured response for the selected inputs. Thirty one center frequencies uniformly spaced from 150.0 to 300.0 Hz were used to update the six dominant design variables. The nominal rms error for the six design variables was 7.40%. The design variable weighting factor was set to  $\alpha = 0.01$ . The weighting matrices for the design variables,  $W_d$ , and the PSD errors,  $W_e$ , were initially set to identity matrices. A variety of averaging bands were used to update the model. A 1% averaging band produced the smallest rms design variable error of 0.780%, but the convergence was slow. The weighting on design variables 7 and 8 was then changed from 1.0 to 0.1 due to the fact that they are significantly less sensitive than the others. With this weighting, the update converged in five iterations to an rms design variable error of 0.178%. The updated model essentially duplicated the simulated test response.

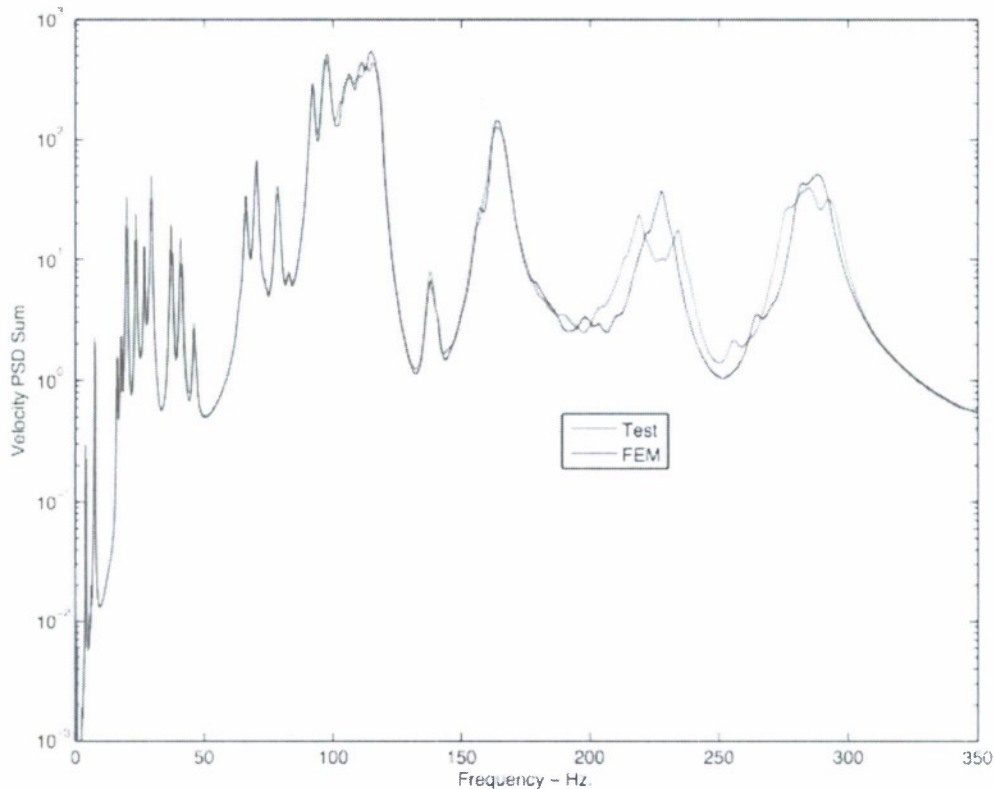


Fig. 8. Test and nominal FEM velocity PSD sums for GSC.

The results generated from this project will serve as a basis for future work that will investigate the use of the technique on larger, more representative model updating applications possessing larger errors in the model parameters. In addition, work will be performed in an attempt to understand how to select the correct averaging band. It is believed that the averaging band can be linked to the modal uncertainty in the nominal model.

### 3.0 CONCLUSION

This research project has produced a complete finite element model validation procedure for the mid-frequency range, or any system with high modal density. The procedure includes sensor placement, analytical model reduction, test-analysis correlation, and finite element model updating. The new approach is totally independent of modal analysis. The results of this research are significant to the Air Force because current state-of-the-art techniques based on modal properties do not work for high-performance spacecraft in the mid-frequency range due to the high modal density. This research is unique because no work has been done on the model validation process for the mid-frequency range. The new approach differs from past work on model updating in the frequency domain because it is based on the principal components from the principal decomposition of the system frequency response. This set of basis vectors is less sensitive to modeling errors and uncertainties, and can be truncated based on system energy. Therefore, the new validation approach directly addresses the high modal density and the sensitivity to uncertainty problems associated with the mid-frequency range.

### 4.0 REFERENCES

Nimityongskul, S., and Kammer, D. C., "Frequency Response Based Sensor Placement for the Mid-Frequency Range", 26<sup>th</sup> International Modal Analysis Conference, Orlando, FL, Feb. 2008. Accepted by *Mechanical Systems and Signal Processing*.

Kammer, D. C., and Nimityongskul, S., "Energy Based Comparison of Test and Analysis Response in the Frequency Domain", 26<sup>th</sup> International Modal Analysis Conference, Orlando, FL, Feb. 2008. Submitted to *Journal of Vibration and Acoustics*.

Nimityongskul, S., and Kammer, D. C., "Frequency Domain Model Reduction Based on Principal Component Analysis", 27<sup>th</sup> International Modal Analysis Conference, Orlando, FL, Feb. 2009. Submitted to *Mechanical Systems and Signal Processing*.

Kammer, D. C., and Nimityongskul, S., "Frequency Band Averaging of Spectral Densities for Updating Finite Element Models", 27<sup>th</sup> International Modal Analysis Conference, Orlando, FL, Feb. 2009. Accepted by *Journal of Vibration and Acoustics*.



## APPENDIX

This Appendix contains listings of the main Matlab computer algorithms developed during the course of this project. Documentation on the use of the computer codes is listed in the files themselves. Table A-1 lists function "xcovave". This function performs frequency band averaging for a set of data. Table A-2 list2 function "xxcorl8". This function performs frequency band correlation on a set of test and analysis frequency response based data. The data can be frequency response from a single input, or spectral densities. Table A-3 lists function "xFBAupdatemx". This function performs the update of the finite element model design variables. Table A-4 lists function "xFBAMin2". This function is called by "xFBAupdatemx" to minimize the cost function.